We will use angle θ to denote the location of the pebble on the hemisphere. The pebble starts at rests on the top the hemisphere and when it is at position θ it has speed v. From energy conservation:

$$2MgR = 2MgR\cos\theta + \frac{1}{2}2Mv^2$$

Therefore

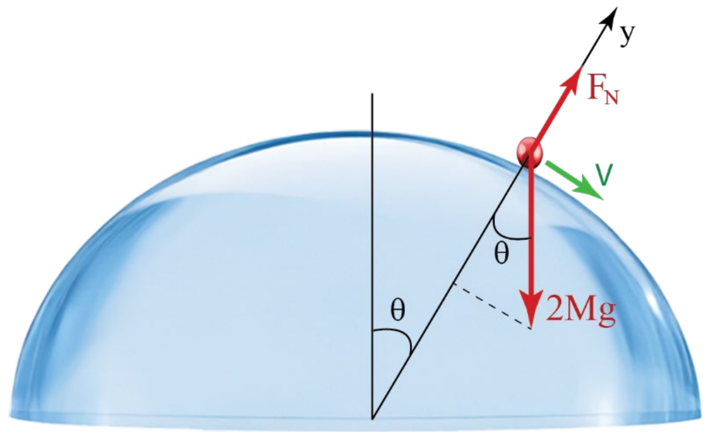

Figure 1

$$2M\frac{v^2}{R} = 4Mg(1-\cos\theta) \tag{1}$$

Now let's consider the forces on the pebble located at position θ. These are the weight of the pebble and the normal force F_N by the hemisphere. Consider the projections of these forces on the radial direction (y-components, see Figure 1):

$$F_N - 2Mg\cos\theta = F_{net} = 2Ma_c = -2M\frac{v^2}{R} \tag{2}$$

From (2) and (1) the magnitude of the normal force:

$$F_N = 2Mg\cos\theta - 2M\frac{v^2}{R} = 2Mg(3\cos\theta - 2) \tag{3}$$

Interestingly, we see that $F_N = 0$ when $\cos\theta = 2/3$. This means that when the position of the pebble is $\theta = cos^{-1}(2/3)$ the pebble loses contact with the hemisphere and flies off its surface.

We now consider the forces that act on the hemisphere (see Figure 2). These are the force of gravity Mg, normal force by the table F_{NT}, the force by the pebble F_P, and the force of friction F_{FR}. The force by the pebble is the exact opposite of the normal force (3) by the hemisphere on the pebble ($\vec{F}_P = -\vec{F}_N$, Newton's 3rd Law).

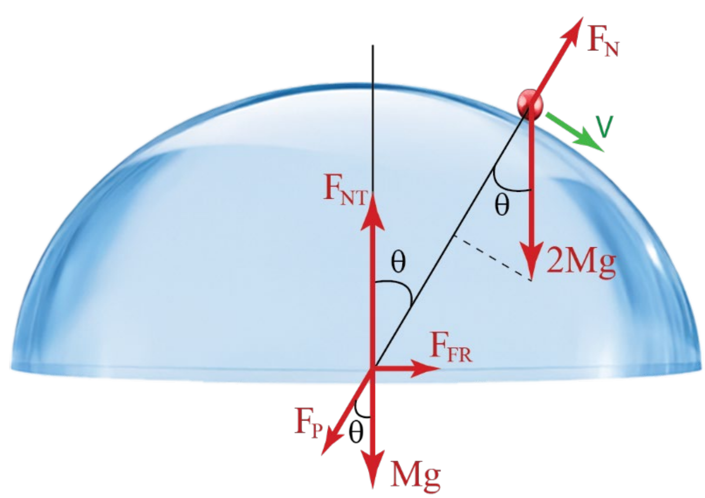

Figure 2

For the vertical components we have:

$$F_{NT} - Mg - F_P \cos\theta = 0$$

Express the normal force F_{NT} of the table on the hemisphere (and use (3) for the magnitude of F_P):

$$F_{NT} = Mg + 2Mg(3\cos\theta - 2)\cos\theta = Mg(1 - 4\cos\theta + 6\,cos^{\,2}\theta) \tag{4}$$

Now consider horizontal components of the forces on the hemisphere:

$$F_{FR} - F_P \sin\theta = 0$$

Assuming the static force of friction is at its maximum possible value $F_{FR} = \mu_S F_{NT}$ and substituting the magnitude of F_P from (3) we have

$$\mu_S F_{NT} = 2Mg(3\cos\theta - 2)\sin\theta$$

Using F_{NT} from (4) we have the expression for the minimum μ_S for the hemisphere to be at rest when the pebble is at position θ:

$$\mu_S = \frac{2Mg(3\cos\theta - 2)\sin\theta}{Mg(1 - 4\cos\theta + 6\,cos^{\,2}\theta)} = 2\frac{3\cos\theta\ \sin\theta - 2\sin\theta}{1 - 4\cos\theta + 6\,cos^{\,2}\theta} \tag{5}$$

To find the value of μ_S that ensures the hemisphere is at rest as the position of the pebble changes from $\theta = 0$ (the starting position) to $\theta = cos^{-1}(2/3)$ (when the pebble flies off the hemisphere) we need to find the maximum of μ_S given by (5). Take derivative of (5) and simplify

$$\frac{d\mu_S}{d\theta} = 2\frac{24\cos^2\theta - 26\cos\theta + 5}{(1 - 4\cos\theta + 6\,cos^{\,2}\theta)^2}$$

$\frac{d\mu_S}{d\theta} = 0$ when $24\cos^2\theta - 26\cos\theta + 5 = 0$. Hence

$$\cos\theta = \frac{26 \pm \sqrt{26^2 - 4\cdot 24\cdot 5}}{2\cdot 24} = \frac{13 \pm 7}{24}$$

Hence the maximum μ_S is reached when $\cos\theta = (13+7)/24 = 5/6$.

Substituting into (5) $\cos\theta = 5/6$ and $\sin\theta = \sqrt{1-(5/6)^2} = \sqrt{11}/6$ yields

$$\mu_S = 2\frac{3\cdot(5/6)\cdot(\sqrt{11}/6) - 2(\sqrt{11}/6)}{1 - 4\cdot(5/6) + 6\cdot(25/36)} = \frac{\sqrt{11}}{11}$$