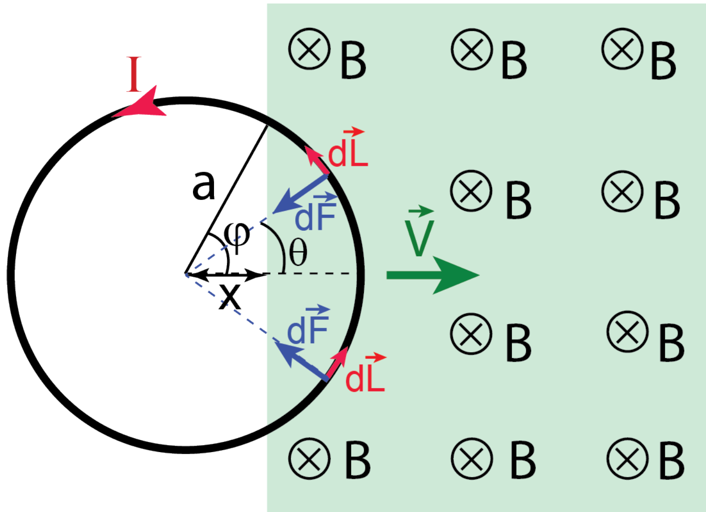
Consider an instant when the ring partially overlaps with the magnetic field region (see Figure). At this instant, the speed of the ring is V, and the center of the ring is at a distance x from the magnetic field region border ($a < x < a$).



During time dt the ring will move distance Vdt. The flux of the magnetic field through the ring will increase by $d\Phi = BdA$ where dA is the area of the strip of the height $2\sqrt{a^2 - x^2}$ and width Vdt:

$$d\Phi = 2BV\sqrt{a^2 - x^2}dt$$

The magnitude of the EMF induced in the ring (Faraday's Law):

$$|\varepsilon| = \left|\frac{d\Phi}{dt}\right| = 2BV\sqrt{a^2 - x^2}$$

The current in the ring (in counterclock direction) at this instant is:

$$I = \frac{\varepsilon}{R} = \frac{2BV}{R}\sqrt{a^2 - x^2}$$

The magnitude of the force $d\vec{F}$ on the current element $d\vec{L}$ ($dL = a \cdot d\theta$) in the magnetic field:

$$dF = \left|I\, d\vec{L} \times \vec{B}\right| = IBa\, d\theta$$

$d\vec{F}$ is directed towards the center of the circle. The "vertical" components of the forces on symmetrical ring elements cancel, and we are left with only "horizontal" components:

$$dF_x = dF \cdot cos\theta = IBa\, cos\theta\, d\theta$$

The net force on the ring is thus:

$F = IBa \int_{-\varphi}^{\varphi} cos\theta\, d\theta$ where $\varphi = arcsin\left(\frac{\sqrt{a^2 - x^2}}{a}\right)$

$$F = 2IBa\, sin\varphi = 2IB\sqrt{a^2 - x^2}$$

Since the current is $I = \frac{2BV}{R}\sqrt{a^2 - x^2}$ we have:

$$F = \frac{4B^2V}{R}(a^2 - x^2)$$

The direction of $\vec{F}$ is opposite to $\vec{V}$. By Newton's Second Law:

$$M\frac{dV}{dt} = -\frac{4B^2V}{R}(a^2 - x^2)$$

or

$$dV = -\frac{4B^2}{MR}(a^2 - x^2)Vdt$$

Since x decreases as the ring moves to the right $dx = -Vdt$ and

$$dV = \frac{4B^2}{MR}(a^2 - x^2)dx$$

The change in the speed of the ring as it moves in the magnetic field entirely is:

$$\Delta V = \int_a^{-a} \frac{4B^2}{MR}(a^2 - x^2)dx = \frac{4B^2}{MR}\left(a^2x - \frac{1}{3}x^3\right)\Big|_a^{-a} = -\frac{16B^2a^3}{3MR}$$